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TESTE 2 - QUESTÕES VARIADAS

26/05/2020

P1

P1.1

F. Custo a minimizar

$$J = \frac{1}{2} \sum_{i=1}^N [z(t_i) - a_0 - a_1 t_i - a_2 t_i^2 - a_3 \sin t_i e^{-t_i}]^2$$

P1.2

Calcular as derivadas parciais de J em ordens a a_0, a_1, a_2, a_3 e igual a 0!

$$\frac{\partial J}{\partial a_0} = 0 \Leftrightarrow \left[- \sum z(t_i) - a_0 - a_1 t_i - a_2 t_i^2 - a_3 \sin t_i e^{-t_i} \right]$$

$$\Leftrightarrow \begin{bmatrix} 1 & \sum t_i & \sum t_i^2 & \sum \sin t_i e^{-t_i} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \sum z(t_i)$$

$$\frac{\partial J}{\partial a_1} = 0 \Leftrightarrow \left[\sum t_i \sum t_i^2 \sum t_i^3 \sum t_i \sin t_i e^{t_i} \right] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \sum z(t_i)$$

$$\frac{\partial J}{\partial a_2} = 0 \Leftrightarrow \left[\sum t_i^2 \sum t_i^3 \sum t_i^4 \sum t_i^2 \sin t_i e^{t_i} \right] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \sum z(t_i) t_i^2$$

$$\frac{\partial J}{\partial a_3} = 0 \Leftrightarrow \left[\sum \sin t_i e^{-t_i} \sum t_i \sin t_i e^{-t_i} / \sum t_i^2 \sin t_i e^{-t_i} / \sum \sin^2 t_i e^{-2t_i} \right] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \sum z(t_i) \sin t_i e^{-t_i}$$

em forma matricial,

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$$\left[\begin{array}{c|c} \sum t_i & \sum t_i \\ \hline \sum t_i & \sum t_i^2 \\ \hline \sum t_i^2 & \sum t_i^3 \\ \hline -\sum \sin(t_i)e^{-t_i} & \sum t_i \sin(t_i)e^{-t_i} \end{array} \right] \left[\begin{array}{c} a_0 \\ a_1 \\ a_2 \\ a_3 \end{array} \right] = \left[\begin{array}{c} \sum z(t_i) \\ \sum t_i z(t_i) \\ \sum t_i^2 z(t_i) \\ \sum z(t_i) x \\ \sin(t_i)e^{-t_i} \end{array} \right]$$

$\underbrace{\qquad\qquad\qquad}_{M} \qquad\qquad\qquad a = b$

P2

$$V_{k+1} = -aV_k^2 + bU_k + V_k$$

Custo

$$J = \frac{1}{2} \sum [V_{k+1} - V_k + aV_k^2 - bU_k]^2$$

equações

$$\frac{\partial J}{\partial a} = 0$$

$$\sum [V_{k+1} - V_k + aV_k^2 - bU_k] V_k^2 = 0$$

$$\frac{\partial J}{\partial b} = 0$$

$$\sum [V_{k+1} - V_k + aV_k^2 - bU_k] U_k = 0$$

3

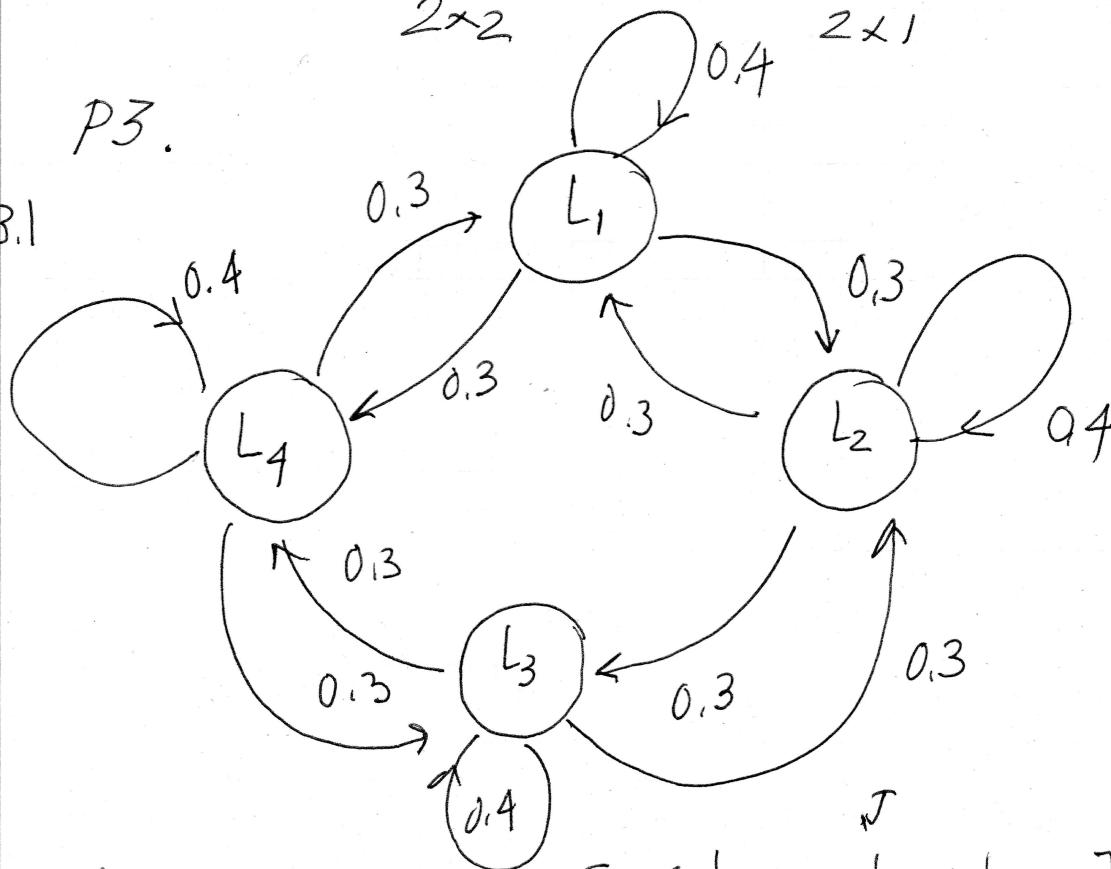


$$\begin{bmatrix} -\sum v_k^4 & \sum u_k v_k^2 \\ \sum v_k^2 u_k & \sum v_k^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum v_{k+1} v_k^2 - \sum v_k^3 \\ \sum v_{k+1} u_k - \sum v_k u_k \end{bmatrix}$$

2×2 2×1 2×1

P3.

P3.1

Matriz de
Transição $A = i$

0.4	0.3	0	0.3
0.3	0.4	0.3	0
0	0.3	0.4	0.3
0.3	0	0.3	0.4



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P3.2

Verificar que $A^2 > 0$

Simples!

$$\begin{bmatrix} \times & \times & 0 & \times \\ \times & \times & \times & 0 \\ 0 & \times & \times & \times \\ \times & 0 & \times & \times \end{bmatrix}$$

$$\begin{bmatrix} \times & \times & 0 & \times \\ \times & \times & \times & 0 \\ 0 & \times & \times & \times \\ \times & 0 & \times & \times \end{bmatrix} \cdot = \begin{bmatrix} \times & \times & \times & \times \\ & \swarrow & & \\ & & \times & \times & \times \end{bmatrix} \checkmark$$

P3.3

$$P^* A^T = P^{*T}$$

$$\Leftrightarrow [P_1^* \quad P_2^* \quad P_3^* \quad P_4^*] = [P_1^* \quad P_2^* \quad P_3^* \quad P_4^*] A$$

$$\Leftrightarrow [P_1^* \quad P_2^* \quad P_3^* \quad P_4^*] =$$

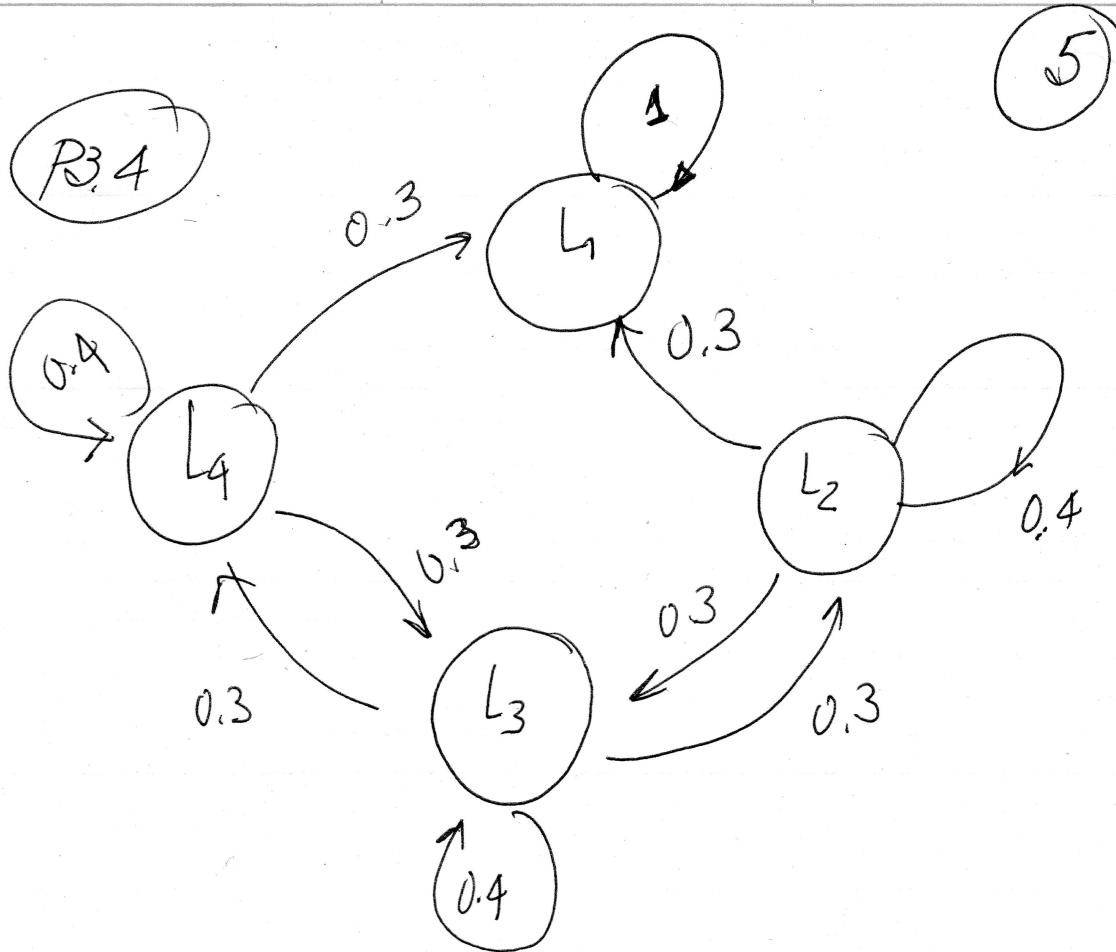
$$\left[0.4P_1^* + 0.3P_2^* + 0.3P_4^* \right]$$

$$\left[0.3P_1^* + 0.4P_2^* + 0.3P_3^* \right]$$

$$\left[0.3P_2^* + 0.4P_3^* + 0.3P_4^* \right]$$

$$\left[0.3P_1^* + 0.3P_3^* + 0.4P_4^* \right]$$

e ... $\boxed{P_1^* + P_2^* + P_3^* + P_4^* = 0}$



⇒

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.3 & 0.4 & 0.3 & 0 \\ 0 & 0.3 & 0.4 & 0.3 \\ 0.3 & 0 & 0.3 & 0.4 \end{bmatrix}$$

Começo em $L_3 \Rightarrow P^T(0) = [0 \ 0 \ 1 \ 0]$

Voo
 $\Rightarrow P^T(1) = P^T(0)A = \underbrace{\begin{bmatrix} 0 & 0.3 & 0.4 & 0.3 \end{bmatrix}}_{\text{não é 'comida'}}$

2º voo $P^T(2) = P^T(1)A =$

$\boxed{2 \times (0.3)^2}$ $2 \times 0.4 \times 0.3 \quad 2 \times (0.3)^2 + (0.4)^2 \quad (0.4)^2 + (0.3)^2]$
 ↳ PROBABILIDADE AO FIM DO 2º VOO.

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$$\begin{pmatrix} u \\ P \end{pmatrix}^u R(4) \begin{pmatrix} v \\ B \end{pmatrix} \rightarrow \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} \text{ ms}^{-1}$$

$$0,1 \text{ rad s}^{-1}$$

$$\Rightarrow \begin{pmatrix} v \\ B \end{pmatrix} = R^T(4) \begin{pmatrix} u \\ P \end{pmatrix} ; \gamma = \theta t = t$$

$$\Rightarrow \begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} \begin{pmatrix} u \\ P \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} -\cos t & -\sin t \\ \sin t & -\cos t \end{bmatrix}$$

Calculo de pressas

$$u - v = x ; r = 1 \text{ rad/s}$$

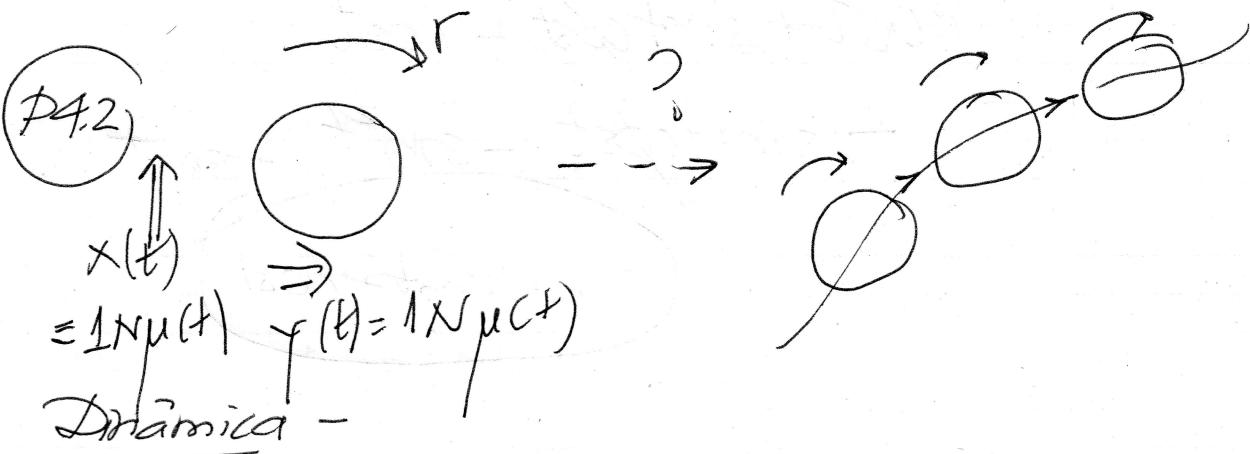
$$v + r u = y$$

$$\Rightarrow (\sin t - \cos t) - (\sin t - \cos t) = 0$$
$$(\cos t + \sin t) + (-\cos t - \sin t) = 0 \quad \checkmark$$

$$\Rightarrow x = y = 0$$

Corpo sem forças aplicadas mantém o vetor velocidade linear!

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$$(1) \ddot{u} - rv = x \quad ; \quad r = \text{rad s}^{-1}$$

$$(2) \dot{v} + ru = y$$

Aplicando T. Laplace,

$$(1) sU(s) - \underbrace{u(0^+)}_{0} - V(s) = \frac{1}{s}$$

$$(2) sV(s) + \underbrace{U(s)}_0 = \frac{1}{s}$$

$$\Rightarrow s^2U(s) = sV(s) + 1$$

$$\Rightarrow s^2U(s) = 1 + \frac{1}{s} - U(s)$$

$$\Rightarrow [s^2 + 1]U(s) = \frac{s+1}{s}$$

$$\Rightarrow U(s) = \frac{1}{s^2 + 1} + \underbrace{\frac{1}{s} \frac{1}{s^2 + 1}}_{\text{Lc}} \frac{1}{s} - \frac{s}{s^2 + 1}$$

$$\Rightarrow U(s) = \frac{1}{s} + \frac{1}{s^2 + 1} - \frac{s}{s^2 + 1}$$

$$\sqrt{\omega^2} = \boxed{[1 + \sin t - \cos t]}$$

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$$u(t) = 1 + \sin t - \cos t$$

$$\Rightarrow (\text{de } \dot{u} - rv = 1) u(t),$$

$$v = \dot{u} - 1$$

$$\Rightarrow \boxed{v = \cos t + \sin t - 1}$$

$$\Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 + \sin t - \cos t \\ \cos t + \sin t - 1 \end{bmatrix}$$

Logo,

$$\overset{u}{\underset{v}{\dot{P}}} = R(u(t)) \begin{bmatrix} u \\ v \end{bmatrix}; u(t) = t$$

$$\Rightarrow \overset{u}{\underset{v}{\dot{P}}} = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix} \begin{bmatrix} 1 + \sin t - \cos t \\ \cos t + \sin t - 1 \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} \cos t + \cos t \sin^2 t - \cos^2 t - \sin t \cos t - \sin^2 t + \sin t \\ \sin t + \sin^2 t - \sin t \cos t + \cos^2 t + \cos t \sin t - \cos t \end{bmatrix} \\ &= \begin{bmatrix} \cos t + \sin t - 1 \\ \sin t - \cos t + 1 \end{bmatrix} \checkmark \end{aligned}$$